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# From UV/IR mixing to closed strings

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Abstract: It was shown in [1] that the leading UV/IR mixing effects in noncommutative gauge theories on D-branes are able to capture information about the closed string spectrum of the parent string theory. The analysis was carried out for D-branes on nonsupersymmetric $\mathbb{C}^3/\mathbb{Z}_N$ orbifolds of Type IIB. In this paper we consider D-branes on twisted circles compactifications of Type II string theory. We find that the signs of the leading UV/IR mixing effects know about the (mass)$^2$ gap between the lowest modes in NSNS and RR closed string towers. Moreover, the relevant piece of the field theory effective action can be reproduced purely in the language of closed strings. Remarkably, this approach unifies in a single structure, that of a closed string exchange between D-branes, both the leading planar and nonplanar effects associated to the absence of supersymmetry.

Keywords: Superstrings and Heterotic Strings, D-branes, Non-Commutative Geometry.




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## Contents



## 1. Introduction

The study of noncommutative field theories has attracted much interest in the last years because noncommutativity is expected to capture basic aspects of the sort distances behavior of gravity [2]. This point of view is supported by the central role that D-branes play in the matrix model description of M-theory [3], since D-branes provide a fundamentally noncommutative description of space-time [4].

In order to explore the consequences of noncommutativity, most works have considered the simple deformation of $\mathbb{R}^n$

$$[x^\mu, x^\nu] = i\theta^{\mu\nu} \,, \tag{1.1}$$

where $\theta^{\mu\nu}$ is an antisymmetric matrix with constant entries. The reason to focus in (1.1) is that explicit calculations can be done for field theories living on such spaces. (1.1) implies uncertainty relations in space-time and therefore the nondecoupling of ultraviolet and infrared degrees of freedom. This new behavior has drastic consequences in the nonplanar sector of field theories: It was shown in [5] that UV/IR mixing leads to the appearance of new infrared divergences in nonplanar graphs, whose origin is in the integration to high momenta in loops.

The relations (1.1) are naturally realized on the world-volume of D-branes in a constant B-field background, where $\theta^{\mu\nu} \sim 1/B_{\mu\nu}$ [6]. Nonplanar field theory diagrams can then be related to nonplanar string diagrams. This suggests an important role of closed strings in the understanding of UV/IR mixing [5, 7]. More generically, it is natural to expect that if (1.1) captures relevant aspects of quantum gravity its effects will know about the closed string sector (although the decoupling of closed strings *does not* fail in the noncommutative field theory limit [8, 9]). In this line, a remarkable relation between UV/IR mixing effects and *properties* of the closed string spectrum was uncovered in [1]. The leading IR divergences due to UV/IR mixing strongly modify the dispersion relations of the theory and in some cases render the perturbative vacuum unstable [5]. A one to one correspondence between these noncommutative instabilities and the presence of closed string tachyons in the parent string theory was obtained for gauge theories on D-branes at nonsupersymmetric $\mathbb{C}^3/\mathbb{Z}_N$ orbifold singularities.

$\mathcal{N}=4$ noncommutative U(1) at finite temperature presents a non-trivial but mild version of UV/IR mixing, where no infrared divergence develops. In spite of that, for temperatures bigger than a critical $T_c$ excitations of tachyonic nature appear in the system [10]. A closely related theory arises on $D3$-branes wrapped along a Scherk-Schwarz circle. It can be seen that noncommutative instabilities set up well below the threshold for the appearance of closed string tachyons, invalidating a direct relation between both phenomena. A motivation of this paper is to determine whether the leading UV/IR mixing effects know about properties of the closed string spectrum in this more general case.

The paper is organized as follows. In section 2 we summarize the results obtained in [1] for noncommutative $\mathbb{C}^3/\mathbb{Z}_N$ quiver theories. In section 3 we consider Type II string theory compactified on twisted circle backgrounds [11], which are generalizations of a Scherk-Schwarz compactification. The closed string spectrum contains tachyons for sufficiently small radius of the circle. In section 3 and 4 we analyze Type IIB $D3$-branes wrapped along the twisted circle. In section 5 we study Type IIA $D2$-branes transverse to the twisted circle. The field theory limit on $D2$-branes and $D3$-branes corresponds to the string regimes with and without closed string tachyons respectively. The sign of the leading UV/IR mixing effects determines if the associated correction will tend to destabilize or not the perturbative vacuum. For the gauge theories on both $D2$- and $D3$-branes we find that these signs are correlated with the $(\text{mass})^2$ difference between the lowest NSNS and RR modes in each winding sector. This will be our proposal on how to generalize the results of [1]. Notice that for $\mathbb{C}^3/\mathbb{Z}_N$ orbifolds the RR zero-point energy is zero and thus the sign of the gap indicates the presence or not of closed string tachyons. However for twisted circle compactifications the mass of closed strings, both in NSNS and RR sectors, is shifted by a nonzero winding energy.

We include in section 2 a proposal for deriving, purely in the language of closed strings, the piece of the gauge invariant effective action responsible for the instabilities [12]. This provides a very direct relation between UV/IR mixing effects and closed strings. A second goal of this paper is to extend this derivation to the gauge theories on $D2$- and $D3$-branes at twisted circle backgrounds. We will show in sections 4 and 5 that this is indeed possible. All the leading UV/IR mixing effects in nonplanar graphs, as well as the leading planar corrections, have the structure of a closed string exchange. The possibility to unify planar and nonplanar effects in a single structure shreds light on the puzzling different behavior of planar and nonplanar sectors. We present our conclusions in section 6. In an appendix we include the derivation of some useful field theory results.



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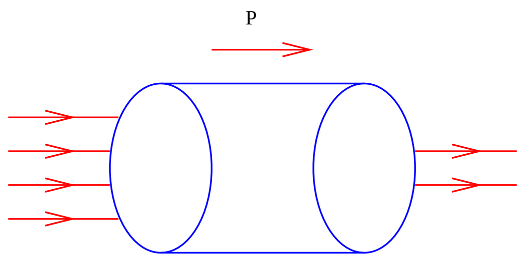


**Figure 1:** Schematic representation of a non-planar graph.

## 2. Open Wilson lines versus closed strings

Non-planar graphs in noncommutative field theories posses an intrinsic UV cutoff in the form of phases whose argument oscillates rapidly for high loop momenta. When the momentum exchange in the non-planar channel vanishes (see figure 1), these phases trivialize and the regularization stops being effective. This implies $\Lambda_{np} \sim \frac{1}{P\theta}$ and, as a consequence, the translation of UV into IR behavior. In its most extreme form, it converts potential UV divergences into IR divergences [5].

The leading IR divergences modify the dispersion relations of noncommutative field theories as follows

$$E^2 = \vec{p}^2 - c\,\frac{g^2}{\tilde{p}^2}\,, \tag{2.1}$$

where $g$ is the (dimensionless) coupling constant, $\tilde{p}^\mu = \theta^{\mu\nu} p_\nu$ and $c$ is a model dependent constant. For non-commutative gauge theories $c \sim N_b - N_f$, with $N_b$ and $N_f$ the number of bosonic and fermionic degrees of freedom in the adjoint representation. Supersymmetric theories have a softer UV behavior and thus the leading IR divergences are absent: $c = 0$ [13]. When $N_b > N_f$, (2.1) turns the low momentum modes unstable [14, 10].

In order to explore a possible connection between noncommutative instabilities and closed string tachyons, it proved useful in [1] to study a family of theories with a sufficiently rich phenomenology. This was provided by noncommutative D-branes on $\mathbb{C}^3/\mathbb{Z}_N$ orbifold backgrounds. The orbifold acts with twist $(a_1, a_2, a_3, a_4)/N$ and $(b_1, b_2, b_3)/N$ on SO(6) spinors and vectors respectively. The integers $a_\alpha$ are subject to $\sum a_\alpha = 0(\mathrm{mod} N)$ and are related to $b_l$ by $b_1 = a_2 + a_3$, $b_2 = a_1 + a_3$, $b_3 = a_1 + a_2$. The gauge theory on $n$ $D3$-branes placed at the fixed point of the orbifold has gauge group $G = \otimes_{i=1}^N \mathrm{U}(n_i)$, where $\sum n_i = n$. The coupling constants of all gauge group factors coincide. The matter content is given by $(\square_i, \overline{\square}_{i+a_\alpha})$ Weyl fermions and $(\square_i, \overline{\square}_{i+b_l})$ complex scalars. Both the gauge theory on the $D3$-branes and the closed string spectrum are supersymmetric if at least one $a_\alpha = 0 \pmod N$.

Turning on a B-field background on two of the spatial directions of the $D3$-branes will render the world-volume noncommutative, i.e. $[x^1, x^2] = i\theta$. In the generic nonsupersymmetric case the noncommutative gauge theory presents a complicated pattern of UV/IR mixing effects. The leading infrared contribution to the nonplanar polarization tensor was calculated in [1]. As can be seen from figure 2, it affects only $\mathrm{U}(1)_i \in \mathrm{U}(n_i)$ degrees of freedom[1] and is non-diagonal in group labels indices. However the linear combinations

[1] In the absence of noncommutativity the theories we are considering have generically mixed anomalies,



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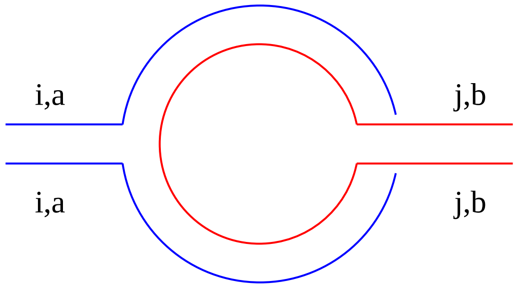


**Figure 2:** Non-planar contribution to the polarization tensor. $i, j$ are group label indices and $a, b$ gauge group indices.

$B_\mu^{(k)} = \frac{1}{\sqrt{N}} \sum e^{2\pi i \frac{jk}{N}} \,\mathrm{Tr}\, A_\mu^{(j)}$ diagonalize it, with the result

$$\Pi_k^{\mu\nu} = \epsilon_k \, \frac{g^2}{\pi^2} \frac{\tilde{p}^\mu \tilde{p}^\nu}{\tilde{p}^4} \,. \tag{2.2}$$

The quantities $\epsilon_k$, which play an analogous role to $c$ in (2.1), have a simple expression in terms of the orbifold twist parameters

$$\epsilon_k = 2 \left( 1 - \sum_{\alpha=1}^{4} \cos \frac{2\pi a_\alpha k}{N} + \sum_{l=1}^{3} \cos \frac{2\pi b_l k}{N} \right) . \tag{2.3}$$

Remarkably, these quantities can be alternatively rewritten in terms of the masses of four low lying modes in the NSNS $k^{th}$ twisted sector

$$\epsilon_k = -16 \prod_{\alpha=1}^{4} \sin \frac{\pi \alpha' m_\alpha^2}{2} \,. \tag{2.4}$$

These masses satisfy the following properties: i) among them is the lowest mode in the NSNS $k^{th}$ twisted sector; ii) $-1 \leq \alpha' m_\alpha^2 < 2$; iii) only one of the $m_\alpha^2$ can be negative (see [1] for details). This implies a direct relation between the sign of $\epsilon_k$ and the presence or not of tachyons in the $k^{th}$ twisted sector of the parent string theory: $\epsilon_k > 0$ if the $k^{th}$ twisted sector contains tachyons; $\epsilon_k = 0$ only if the $k^{th}$ twist is supersymmetry-preserving; $\epsilon_k < 0$ when the twisted sector is nonsupersymmetric but does not include tachyons. This last situation does not arise in $\mathbb{C}^n/\mathbb{Z}_N$ orbifolds with $n < 3$, and was the reason to consider orbifolds of $\mathbb{C}^3$.

Let us come back now to the origin of UV/IR mixing, which is in the uncertainty relations derived from (1.1): $\Delta x^1 \Delta x^2 \geq \theta$ in our case. This phenomenon is present in any noncommutative theory, independently whether there are UV or IR divergences. It would seem then that the natural degrees of freedom of noncommutative theories are those that made this behavior evident. Contrary to this expectation, noncommutative field theories are formulated using local fields. This puzzle is solved by studying the effective action of

that will lead to the massification of the U(1)'s through a Green-Schwarz mechanism [15]. When $\theta \neq 0$ the situation is more subtle: the U(1)'s become massive and decouple only for $\tilde{p} = 0$ [16], while for $\tilde{p} \neq 0$ the anomaly vanishes [17]. We will always consider the latter case.

the theory. It has been shown both for scalar [18] and gauge theories [19, 12, 20] that the 1-loop nonplanar effective action, including contributions from all the N-point functions, can be rewritten in terms of straight open Wilson line operators [21, 22, 23, 24]. This result was extended in [25] to the 2-loop effective action of scalar theories. Straight open Wilson line operators exhibit the desired behavior since their momentum, $p$, is correlated with their transversal extent $\tilde{p}$,

$$\widetilde{W}(p) \;=\; \mathrm{Tr} \int d^4x \; P_* \left( e^{i\,g \int_0^1 d\sigma\, \tilde{p}^\mu A_\mu(x+\tilde{p}\,\sigma)} \right) * e^{ipx} \,. \tag{2.5}$$

For the orbifold gauge theories above, the gauge invariant piece of the 1-loop effective action containing (2.2) has the simple expression [1]

$$\Delta S = \frac{1}{2\pi^2} \sum_{k=0}^{N-1} \epsilon_k \int \frac{d^4p}{(2\pi)^4}\, \frac{1}{\tilde{p}^4}\, W^{(N-k)}(p)\, W^{(k)}(-p) \,, \tag{2.6}$$

where $W^{(k)} = \frac{1}{\sqrt{N}} \sum_{j=1}^{N} e^{2\pi i \frac{jk}{N}} \widetilde{W}^{(j)}$, and $\widetilde{W}^{(j)}$ denotes (2.5) with the vector field belonging to the $j^{th}$ gauge group factor.

Several facts suggest the interpretation of (2.6) in terms of a closed string exchange between D-branes. (2.6) seems to know about the different closed string twisted sectors since the quantities $\epsilon_k$ measure an independent property of each sector. In addition, closed string modes in the $k^{th}$ twisted sector couple to linear combinations of field theory operators such as those that define $B^k_\mu$ and $W^k$ [26]. Moreover, it has been shown in [27, 28, 29] that closed strings couple to straight Wilson line operators on noncommutative D-branes. We should notice that closed strings can only couple to gauge invariant operators and the open Wilson lines provide the basic set of such operators in noncommutative gauge theories [24]. However, the condition of gauge invariance only determines the separation between the start and end point of the Wilson line but not the path.

Reviewing [12, 1], we will propose now a direct derivation of (2.6) in terms of a closed strings. Our motivation is twofold. We have mentioned two aspects of UV/IR mixing, the first associated to the physics of the non-planar sector and the appearance of an extreme infrared behavior, the second associated to the question of what are the natural noncommutative excitations and the role of open Wilson line operators. Closed strings provide a connection between these two aspects since they couple naturally to open Wilson lines and, as we will see, the IR divergent term $1/\tilde{p}^4$ in (2.6) can be related to a closed string propagator. Our second goal is to obtain an alternative understanding of the $\epsilon_k$'s and its relation to closed string tachyons.

In the absence of B-field, scalar closed string modes couple to the brane tension at leading order in $\alpha'$: $T \sim \mathrm{Tr}\,\mathbb{1}/\alpha'^2$. When $B \neq 0$ the trivial field theory operator $\mathrm{Tr}\,\mathbb{1}$ gets promoted to the open Wilson line operator (2.5). Hence, at leading order in $\alpha'$, the contribution to the $D3$-brane effective action from the emission, propagation and posterior absorption of a scalar closed string mode $\varphi$ in the $k^{th}$ twisted sector is

$$\Delta S = |D_\varphi|^2 \int \frac{d^4p}{(2\pi)^4}\, W^{(N-k)}(p) W^{(k)}(-p)\, f(\tilde{p}, u) \,, \tag{2.7}$$

with $D_\varphi$ a numerical factor proportional to the disk amplitude of $\varphi$ with no open string insertion. The function $f(\tilde{p}, u)$ denotes the closed string propagator

$$f(\tilde{p}, u) = \alpha'^{-\frac{d+2}{2}} \int \frac{d^d v}{(2\pi)^d} \frac{e^{ivu}}{v^2 + \tilde{p}^2 + (2\pi\alpha' m_\varphi)^2} \,. \tag{2.8}$$

$d$ is the number of dimensions transverse to the D-brane where the twisted field $\varphi$ can propagate: $d = 0, 2, 4, 6$ depending on the particular $\mathbb{C}^3/\mathbb{Z}_N$ orbifold. We have defined $v = 2\pi\alpha' p_\perp$, with $p_\perp$ the transversal momentum to the D-brane. $u$ has been introduced in order to have a well defined closed string propagator, it has the interpretation of an infrared regulator from the point of view of the field theory. In the denominator we have used the relation between open ($\eta$) and closed ($g$) string metrics $g^{-1} = \eta^{-1} - \theta\eta\theta/(2\pi\alpha')^2$ [30], and discarded terms suppressed by two $\alpha'$ powers; $m_\varphi$ is the mass of $\varphi$. The factor of $\alpha'$ in front of the integral can be obtained just by dimensional analysis.

From (2.7) we want to extract a contribution to the noncommutative field theory effective action. We need to take the limit $\alpha' \to 0$. In this limit $f$ diverges due to the negative $\alpha'$ power in front of the integral. However we should notice the following. If we had done the $\alpha' \to 0$ limit of the annulus diagram associated to each nonplanar N-point function, we would had obtained a result $\mathcal{O}(\alpha'^0)$ whose leading IR contribution should reproduce the corresponding term in (2.6). This has been checked for the 2-point function in [31, 8]. The question is then how to directly define an $\mathcal{O}(\alpha'^0)$ contribution from (2.7), regarding as artifacts other $\alpha'$ powers. We observe that $\tilde{p}$ acts as an infrared regulator for the integral in (2.8). Therefore, for $\tilde{p} \neq 0$, we can expand the integral in powers of $(2\pi\alpha' m_\varphi)^2 \sim \alpha'$ to the desired order. At $\mathcal{O}(\alpha'^0)$ we obtain

$$f(\tilde{p}, u)|_{\mathcal{O}(\alpha'^0)} \sim \int \frac{d^d v}{(2\pi)^d} \frac{e^{ivu}}{(v^2 + \tilde{p}^2)^{\frac{d}{2}+2}} \to_{u\to 0} \frac{c}{\tilde{p}^4} \,, \tag{2.9}$$

where $c$ is a constant that depends on the dimension $d$ and whose value we will not need to determine. After removing the field theory infrared regulator $u$ we recover $1/\tilde{p}^4$, independently of $d$. The possibility to relate $1/\tilde{p}^4$ to a closed string propagator does not mean that the decoupling of closed strings fails in the noncommutative field theory limit, since the IR singularities do not have kinetic part. Hence they do not force the introduction of additional degrees of freedom.

In (2.7) we have considered the exchange of a single closed string mode. Any closed string mode able to couple to $W^k$ will contribute the same $1/\tilde{p}^4$ up to a numerical factor depending on $D_\varphi$ and $m_\varphi$. The coefficients $\epsilon_k$ that appear in (2.6) will be thus a collective effect of the closed string towers. However, we know from (2.4) that $\epsilon_k$ is determined by a finite set of low lying string modes and, moreover, its sign just depends on the presence of tachyons. To reconcile these two facts, notice that modes in the NSNS and RR sectors contribute with opposite sign to the exchange between D-branes. In addition, when the twisted sector is supersymmetric we have $\epsilon_k = 0$, implying that the contribution from both towers must cancel. This suggests to consider $\epsilon_k$ as a measurement of the misalignment between the NSNS and RR towers. It seems then consistent that the presence of tachyons, which can only belong to the NSNS sector, is linked to the sign of the mentioned misalignment given that the lightest RR modes are massless.

We would like to end this section with a comment on the cases with scalars in the adjoint representation (this situation only arises if some $b_l = 0$). It was shown in [27, 28, 29] that in those cases, closed string modes couple to generalized Wilson line operators

$$\widetilde{W}(p,v) = \mathrm{Tr} \int d^4x \; P_* \left( e^{i\,g \int_0^1 d\sigma (\, \tilde{p}^\mu A_\mu (x+\tilde{p}\,\sigma) + v_a \,\phi_a (x+\tilde{p}\sigma))} \right) * e^{ipx} \,, \tag{2.10}$$

where $v_a$ label as before the momentum in the directions transverse to the $D3$-branes and $\phi_a$ denote the adjoint (real) scalars. In this case the closed string derivation of the effective action we have proposed suggests to extent (2.6) to

$$\Delta S = \frac{1}{2\pi^2 c} \sum_{k=0}^{N-1} \epsilon_k \int \frac{d^4p}{(2\pi)^4} \frac{d^dv}{(2\pi)^d} \; W^{(N-k)}(p,v)\, W^{(k)}(-p,-v) \frac{e^{ivu}}{(v^2+\tilde{p}^2)^{\frac{d}{2}+2}} \,, \tag{2.11}$$

with $c$ the same constant as in (2.9). Adjoint scalars, as gauge bosons, get pole-like IR corrections to their self-energy. They can be obtained from (2.11) by expanding $W^k$ to linear order in $\mathrm{Tr}\,\phi_a$, using[2]

$$\int \frac{d^dv}{(2\pi)^d} \frac{e^{ivu}}{(v^2+\tilde{p}^2)^{\frac{d}{2}+2}} \; v_a v_b \rightarrow_{u\rightarrow 0} \frac{c}{2\tilde{p}^2} \delta_{ab} \,. \tag{2.12}$$

The relative factor 1/2 between (2.9) and (2.12) is important to correctly reproduce the field theory results, as we will see later. This support the usefulness of the closed string point of view to understand UV/IR mixing effects, since the generalization of the open Wilson line operators to include the adjoint scalars (2.10) is not dictated by gauge invariance.

## 3. Strings on twisted circles

In this and the following sections we will explore whether the remarkable connection between UV/IR mixing and closed strings found for D-branes at $\mathbb{C}^3/\mathbb{Z}_N$ orbifolds extends to more general cases. We will consider D-branes in Type II string theory compactified on twisted circle backgrounds. These are $\mathbb{R}^9 \times S^1$ spacetimes where shifts along the circle are combined with rotations on several 2-planes [11]. We will focus on the case where the rotations act on three planes and coincide with those that define a $\mathbb{C}^3/\mathbb{Z}_N$ orbifold

$$(y, z^l) \rightarrow \left( y + 2\pi R, e^{2\pi i \frac{b_l}{N}} z^l \right) , \tag{3.1}$$

where $y$ denotes the coordinate along the circle, $z^l$ the complex coordinates of $\mathbb{C}^3$ and $b_l$ are the twist parameters of the previous section. For $R \rightarrow \infty$ we recover Type II string theory on $\mathbb{R}^{10}$. For $R = 0$ the Type IIA(B) twisted circle compactification reduces, after T-duality, to the associated $\mathbb{C}^3/\mathbb{Z}_N \otimes \mathbb{R}^4$ Type IIB(A) orbifold model [32].

The spectrum of closed strings on these backgrounds was derived in [11]. World-sheet bosons and fermions with indices transverse to the three 2-planes behave as in an ordinary circle compactification, except for a different quantization condition on $p_y$. Invariance of

[2]The integral (2.9) for $d = 2, 4, 6$ can be performed explicitly resulting in all cases proportional to $\frac{u^2}{\tilde{p}^2} K_2(u\tilde{p})$, with $K_2$ a modified Bessel function. Using the properties of Bessel functions, it is now easy to derive (2.12).

the wavefunctions under (3.1) requires

$$p_y = \frac{1}{R}\left(m - \sum_l \frac{b_l J_l}{N}\right), \qquad m \in \mathbb{Z}\,, \tag{3.2}$$

where $J_l$ are the angular momenta on the three 2-planes. The identification (3.1) links the winding number along the circle, $w$, with the boundary conditions for the world-sheet fields with indices along the 2-planes. For a given $w$, and for the restricted set of examples we are considering, the left and right moving modes of these fields behave as those in the $k^{th}$ twisted sector of the associated $\mathbb{C}^3/\mathbb{Z}_N$ orbifold, with $k = w(\mathrm{mod}N)$. It is useful to define

$$N'_k = N_k - c_k\,, \qquad \bar{N}'_k = \bar{N}_k - \bar{c}_k\,, \tag{3.3}$$

with $N_k$ ($\bar{N}_k$) the oscilator contribution to $L_0$ ($\bar{L}_0$) and $c_k$ ($\bar{c}_k$) the zero-point energy of the left (right) modes in the NS or R $k^{th}$ twisted sector of the orbifold model. The mass spectrum of closed strings in the twisted circle background (3.1) is then given by

$$M^2 = \frac{2}{\alpha'}(N'_k + \bar{N}'_k) + \frac{1}{R^2}\left(m - \sum_l \frac{b_l J_l}{N}\right)^2 + \frac{w^2 R^2}{\alpha'^2}\,, \tag{3.4}$$

subject to the level matching constrain

$$N'_k - \bar{N}'_k = w\left(m - \sum_l \frac{b_l J_l}{N}\right). \tag{3.5}$$

$\mathbb{C}^3/\mathbb{Z}_N$ orbifolds generically have tachyons in the twisted sectors ($k \neq 0$). Therefore the associated twisted circle backgrounds may also have tachyons for appropriate radius, provided that $w \neq 0(\mathrm{mod}N)$. Since the second term in (3.4) always gives a positive contribution to the mass, let us consider the case it vanishes. This implies: i) the level matching condition reduces to that of the related $\mathbb{C}^3/\mathbb{Z}_N \otimes \mathbb{R}^4$ orbifold model; ii) $\sum_l \frac{b_l J_l}{N} \in \mathbb{Z}$, which imposses the orbifold projection. Under these conditions, the first term in (3.4) reproduces the masses of the orbifold $k^{th}$ twisted sector. The lowest mode in each winding sector of the twisted circle compactification is contained among this set. When $m_k^2 < 0$, with $m_k$ the mass of the lowest mode in the $k^{th}$ twisted sector of the associated orbifold,[3] the twisted circle compactification will contain tachyons for radii

$$\frac{R}{\sqrt{\alpha'}} < k^{-1}\sqrt{\alpha' |m_k^2|}\,, \tag{3.6}$$

where we have taken $w = k$ in order to minimize the winding energy.

We will be interested in studying field theories arising on noncommutative D-branes in these backgrounds. We will consider Type IIB $D3$-branes wrapped along the twisted circle and Type IIA $D2$-branes transverse to the twisted circle [33, 34]. The intrinsic scale of the $D3$-brane theory is set by the Kaluza-Klein masses $m \sim 1/R$. Hence the natural field

[3]See [1] for the expression of $m_k^2$ in terms of the twist parameters $b_l$.

| $A^{ij}_{\mu\nu}$ | $\psi^{ij}_{\alpha}$ | $\phi^{ij}_{l}$ |
|---|---|---|
| $\frac{1}{R}\left(m-\frac{i-j}{N}\right)$ | $\frac{1}{R}\left(m-\frac{i-j+a_\alpha}{N}\right)$ | $\frac{1}{R}\left(m-\frac{i-j+b_l}{N}\right)$ |

**Table 1:** Field content of the theory and associated $p_y$. $A^{ij}_{\mu\nu}$ are vector fields, $\psi^{ij}_{\alpha}$ ($\alpha = 1, \ldots, 4$) Weyl fermions and $\phi^{ij}_{l}$ ($l = 1, 2, 3$) complex scalars. $a_\alpha$ are the $\mathbb{C}^3/\mathbb{Z}_N$ twist parameters for SO(6) spinors, as in section 2.

theory limit for the $D3$-branes consist in sending $\alpha' \to 0$ keeping $R$ fixed. In this regime the winding energy dominates (3.4) and, as can be seen from (3.6), there are no tachyonic modes. Contrary, the natural scale of the $D2$-brane theory is $R/\alpha'$, governing the mass of strings with both ends on the brane and non-zero winding along the circle. The interesting field theory limit in this case is achieved by keeping $R/\alpha'$ fixed as $\alpha' \to 0$. From (3.6) we observe that this corresponds to the regime with closed string tachyons. Therefore we obtain an optimal setup to explore the relation between the leading UV/IR mixing effects and properties of the closed string spectrum. Notice however that the noncommutative field theory is not able to resolve the value of $R$ for which tachyonic modes appear, since any $R \sim \sqrt{\alpha'}$ maps to $R = 0$ in the field theory limit appropriate for the $D3$-brane and $R/\alpha' = \infty$ for the $D2$-brane.

## 4. Wrapped $D3$-branes

In this section it will be convenient to view the previous twisted circle backgrounds as freely acting orbifolds of $\mathbb{R}^9 \times S^1_{R'}$, with $R' = NR$. We denote the identification (3.1) by $t$; it satisfies $t^N = 1$.

We place $n$ $D3$-branes on $\mathbb{R}^9 \times S^1_{R'}$ wrapped along the circle and localized at $z^l = 0$. The gauge theory on the $D3$-branes is $\mathcal{N} = 4$ U($n$). After performing the orbifold projection, only modes whose wavefunction is left invariant survive

$$Z = \gamma_t \,(t\, Z)\, \gamma_t^{-1}\,, \tag{4.1}$$

where $Z$ is any of the $\mathcal{N} = 4$ fields. The matrix $\gamma_t$ represents the action of the orbifold on the Chan-Paton indices. Since it must satisfy $\gamma_t^N = 1$, it can be taken to be diagonal with eigenvalues $e^{2\pi i j/N}$ appearing with multiplicity $n_j$ ($\sum n_j = n$), as for $\mathbb{C}^3/\mathbb{Z}_N$ orbifolds. Let us decompose the field $Z$ in components $Z_{ij}$, transforming as $(\square_i, \overline{\square}_j)$ under the subgroup $\otimes\, \mathrm{U}(n_j)$ which commutes with $\gamma_t$. The condition (4.1) translates into a restriction on the allowed momenta $p_y$ analogous to (3.2)

$$p_y = \frac{1}{R}\left(m - \frac{i - j + \sum_l b_l J_l}{N}\right), \qquad m \in \mathbb{Z}\,. \tag{4.2}$$

Hence the theory on the wrapped $D3$-branes has the degrees of freedom of $\mathcal{N} = 4$ U($n$) compactified on a circle of radius $R$, with both supersymmetry and gauge symmetry broken by the different fractional momenta along the circle; see table 1.

We turn on now a constant B-field along the two non-compact spatial directions of the $D3$-brane, rendering its world-volume noncommutative. Since the theory behaves at high energies as $\mathcal{N}=4$ Yang-Mills, there will be no UV divergence and thus no associated IR divergence. But given that supersymmetry is broken at the scale $1/NR$, we expect finite UV/IR mixing effects [10]. From now on we will take the point of view of the dimensionally reduced theory and will focus on the zero-modes. The only fields with zero-modes on the circle are: $A_\mu^{ii}$, the gauge fields associated to $\otimes\,\mathrm{U}(n_i)$, $\psi_\alpha^{ii+a_\alpha}$ and $\phi_l^{ii+b_l}$. This is the same set of fields that would survive the associated $\mathbb{C}^3/\mathbb{Z}_N$ orbifold projection. A complete treatment should of course include all fields and their Kaluza-Klein modes. The reason to focus on zero-modes is that the leading UV/IR mixing terms are simpler, and we expected them to have an stronger effect, on the massless 3d fields.

As we have already mentioned, the nonplanar 2-point functions project on U(1) degrees of freedom and mix different group labels (see figure 2). We normalize the Lie algebra generators of each $\mathrm{U}(n_i)$ such that $\mathrm{Tr}\, t_r t_s = \delta_{rs}$, with $t_0 = \frac{1}{\sqrt{n_i}}\mathbb{1}$. We take the external legs of the nonplanar 2-point functions to correspond to $\mathrm{Tr}\, A_\mu^{ii} = A_\mu^{ii\,(0)}\sqrt{n_i}$, and analogously for adjoint scalars.[4] It is easy to see that with this choice of normalization for the external legs, the nonplanar 2-point functions are independent of the ranks $n_i$. We can then proceed with the evaluation as if our gauge group would be $\mathrm{U}(1)^N$. The 2-point functions for the zero-modes in that case are calculated in detail, at leading order, in an appendix.

Let us introduce some convenient notation. We define $\Pi_{ij}^{(1)}$ by $\Pi_{ij}^{\mu\nu} = \frac{\tilde{p}^\mu \tilde{p}^\nu}{\tilde{p}^2}\,\Pi_{ij}^{(1)}$, where $\mu = 1,2,3$ are the three noncompact directions, and $\Pi_{ij}^{(2)} = \Pi_{ij}^{yy}$. All other components of the polarization tensor for zero-modes are zero. The matrix $\Pi_{ij}^{(3)}$ will refer to the self-energies of adjoint scalar zero-modes when present (this requires that some $b_l = 0$), $\Sigma_{ij}^{ab} = \delta^{ab}\,\Pi_{ij}^{(3)}$. Using this compact notation and (A.5)-(A.7), the nonplanar 2-point functions are given by

$$\Pi_{ij}^{(M)} = \frac{g^2}{2\pi^2 \tilde{p}}\, c_{ij} \sum_\rho (-1)^{F_\rho} \int_0^\infty dl\; N_{\bar{\nu}_\rho}(Rl)\, f^{(M)}(\tilde{p}l)\,. \tag{4.3}$$

The index $\rho$ runs over all degrees of freedom transforming in the $(\square_i,\bar{\square}_j)$ and $(\square_j,\bar{\square}_i)$ representations, and $F_\rho$ is their fermion number. The exchange of an adjoint field in the loop contributes twice that of a bifundamental, thus $c_{ii} = 1$ while $c_{ij} = 1/2$ for $i \neq j$. The function $N_{\bar{\nu}_\rho}$ is given in (A.2). It represents the generalization of the Bose-Einstein and Fermi-Dirac distributions to fields with general fractional momenta along a circle, $p_y = (m - \bar{\nu}_\rho)/R$. The functions $f^{(M)}$ are

$$f^{(M)}(x) = \sin x + b^{(M)} x \cos x\,, \tag{4.4}$$

with $b^{(1)} = -1$, $b^{(2)} = 1$ and $b^{(3)} = 0$.

[4]We will discard backgrounds with some $a_\alpha = 0$, since the associated orbifold and twisted circle backgrounds never contain closed string tachyons. Hence we will not encounter adjoint spinors with zero-modes.

The integrals in (4.3) can be easily evaluated recalling (A.1) and performing a Poisson resummation

$$\int_0^\infty dl N_{\bar{\nu}_\rho}(Rl)\sin\tilde{p}l = \pi T \sum_{k=0}^{N-1} \cos\left(2\pi\bar{\nu}_\rho k\right)\, N_{\nu_k}(T\tilde{p})\,, \tag{4.5}$$

where we have defined $\nu_k = k/N$ and $T = 1/2\pi RN$. $\int dl N_{\bar{\nu}_\rho} l\cos\tilde{p}l$ can be directly obtained from the above expression, using that $l\cos\tilde{p}l = \frac{\partial \sin\tilde{p}l}{\partial\tilde{p}}$. Substituting these results, together with the explicit expression of $N_{\nu_k}$ (A.2), we arrive at

$$\Pi^{(M)}_{ij} = g^2 T^2 \sum_{k=0}^{N-1} M_{ij;k}\, F^{(M)}_k(T\tilde{p})\,, \tag{4.6}$$

with

$$F^{(M)}(x) = \frac{1}{\cosh 2\pi x - \cos 2\pi\nu_k}\left[\frac{\sinh 2\pi x}{2\pi x} + b^{(M)}\,\frac{1-\cos 2\pi\nu_k\cosh 2\pi x}{\cosh 2\pi x - \cos 2\pi\nu_k}\right], \tag{4.7}$$

and $M_{ij;k} = c_{ij}\sum_\rho (-1)^{F_\rho}\cos 2\pi\bar{\nu}_\rho k$. From the data of Table 1, an straightforward calculation gives

$$M_{ij;k} = \epsilon_k\, \cos\left(2\pi(i-j)\nu_k\right). \tag{4.8}$$

Remarkably the quantities $\epsilon_k$ coincide with those resulting from the diagonalization of the polarization tensor for the $\mathbb{C}^3/\mathbb{Z}_N$ quiver theories analyzed in section 2, eq. (2.3). However the matrices $\Pi^{(M)}_{ij}$ are not yet in a diagonal form. Notice that their $ij$ entry depends only on $(i-j)\,\mathrm{mod}N$. Any matrix with this property can be diagonalized by a set of orthogonal vectors $(e_k)_j = \frac{1}{\sqrt{N}}e^{2\pi i jk/N}$. In the new basis (4.6) reads

$$\Pi^{(M)}_k = g^2 T^2\, N\, \epsilon_k\, F^{(M)}_k(T\tilde{p})\,. \tag{4.9}$$

The sign of the 2-point functions is governed by $\epsilon_k$, as it was the case for the $\mathbb{C}^3/\mathbb{Z}_N$ quiver theories, because the functions $F^{(M)}_k$ are always positive. This can easily be shown by direct inspection of (4.7). For $k\neq 0$, $F^{(M)}_k$ is finite as $\tilde{p}\to 0$. The case $k=0$ is special. $F^{(M)}_0$ diverges for small $\tilde{p}$ since this sector contains the IR divergences of the noncompact 4d theory. However, $\epsilon_0 = 0$ because the uncompatified limit is supersymmetric. As $T\to\infty$, $F^{(M)}_k$ tends to $1/2\pi T\tilde{p}$. In this limit (4.9) reproduces the $1/\tilde{p}$ IR divergences of the $\mathbb{C}^3/\mathbb{Z}_N$ quiver theory on a $D2$-brane. Therefore at some point of the flow to small radius (4.9) will dominate the dispersion relations for low momentum modes, and noncommutative instabilities will appear.

The analysis of the critical radius at which some modes become unstable is quite involved. It strongly depends on which fields we are considering, vector or scalar, on the sector $k$ and on the particular brane configuration, characterized by the ranks $n_i$. We will not go through this complicated phenomenology, because what interests us is the structure behind these effects. However for completion, we include below the leading planar contributions to the 2-point functions.

We reduce again to the zero-modes of gauge bosons and adjoint scalars. As can be seen from figure 3, in this case there is no mixing among the different group labels. The planar correction for the 3-dimensional gauge bosons is absent; for the others fields we have

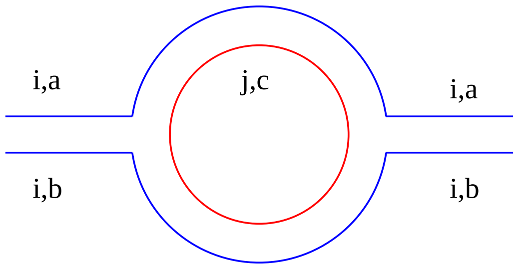


**Figure 3:** Planar contribution to the 2-point function of adjoint fields. $i, j$ are group label indices and $a, b, c$ gauge group indices.

(see (A.8))

$$\hat{\Pi}^{(2)}_{ii} = 2\hat{\Pi}^{(3)}_{ii} = -\frac{g^2}{\pi^2} \sum_\rho c_\rho \, (-1)^{F_\rho} \int_0^\infty dl \, N_{\tilde{\nu}_\rho}(Rl) \, l \, . \tag{4.10}$$

The summation in $\rho$ is over the degrees of freedom transforming in the $(\Box_i,\bar{\Box}_j)$ and $(\Box_j,\bar{\Box}_i)$, for any $j$. $c_\rho = 1$ for a field in the adjoint representation, while $c_\rho = 1/2$ for a bifundamental. The external legs will be normalized now such that $\hat{\Pi}^{(2)}_{ii}$ couples to $\mathrm{Tr}(A^{ii}_y)^2$, which is appropriate for a planar graph, and analogously for $\hat{\Pi}^{(3)}_{ii}$. With this normalization, the 2-point functions will include an explicit dependence on the ranks $n_j$. We obtain

$$\hat{\Pi}^{(2)}_{ii} = -\frac{2g^2T^2}{\pi^2} \sum_{k=0}^{N-1} \epsilon_k \left( \sum_{j=1}^{N} n_j \, \cos\left(2\pi(i-j)\nu_k\right) \right) Z_2[\nu_k] \, , \tag{4.11}$$

with $Z_m[x]$ a generalized Riemann Zeta function.

It is worth stressing that the result of evaluating the integrals in (4.3) could be written again in terms of functions $N_{\nu_k}$, with their argument changed from $Rl$ to $T\tilde{p}$. A similar observation, in the context of noncommutative thermal field theories, was done in [35]. Since $T \sim 1/R$, they suggested that there must be some hidden winding states in the noncommutative theory. In [7] it was argued that these mysterious states were closed strings winding around the temperature circle of a parent string theory. They noticed that an infinite set of closed strings must play a role, very much as in ordinary field theory limits. This raised the question about the usefulness of the closed string picture. In the next section we will present a very explicit realization of these ideas along the lines of section 2. In particular we will show that the closed string picture is indeed useful to understand the structure of both (4.9) and (4.11).

## 5. Closed string exchange

Let us analyze what type of Wilson line operators couple to closed strings in twisted circle backgrounds. It is useful to consider first $D3$-branes wrapped along an ordinary circle of radius $R' = NR$. Closed strings couple to the straight open Wilson line operators (2.10) on noncommutative D-branes. These operators depend on the components of the gauge field along the noncommutative directions and on adjoint scalars, associated to the position of the branes in transverse dimensions. However this must be generalized when one direction on the D-brane is compact. T-dualizing, the $D3$-brane is converted to a $D2$-brane localized on the circle. The zero-mode of $A_y$ describes the positions of branes on the T-dual circle and thus plays a similar role to the adjoint scalars. Hence in order to be consistent with T-duality, we should include a dependence on $A_y$ in the exponent of the Wilson line.

The explicit construction of the open Wilson line operators that are gauge invariant under the 4d gauge transformations is very involved. However we will not need to know their precise expression. We are only interested on the coupling of closed strings to the zero-modes of gauge fields and scalars, since this will be enough to reproduce the results of the past section. With this restriction, we propose the following modification of (2.10)

$$\widetilde{W}_{p_y=0} = 2\pi R' \ \mathrm{Tr} \int d^3x \ P_* \left( e^{i\, g \int_0^1 d\sigma \, (\tilde{p}^\mu A_\mu(x(\sigma)) + v_a \phi_a(x(\sigma)) + 2\pi R' s A_y(x(\sigma)))} \right) * e^{ipx} \ + \cdots \, , \tag{5.1}$$

where $A_\mu(x) = \frac{1}{2\pi R'} \int dy A_\mu(x,y)$ and the same for $A_y$ and $\phi_a$; we have denoted $x(\sigma) = x + \tilde{p}\sigma$. The dots indicate terms depending on modes with $p_y \neq 0$, necessary to make $\widetilde{W}$ 4d gauge invariant. The term written explicitly above is however gauge invariant under 3d gauge transformations. As in section 2, $v = 2\pi\alpha' p_\perp$, with $p_\perp$ the momentum on the transverse directions of the associated closed string mode. $s \in \mathbb{Z}$ is its winding number. Notice that $2\pi R' s = 2\pi\alpha' p_{\tilde{v}}$, with $p_{\tilde{v}} = sR'/\alpha'$ the momentum on the dual T-circle. The factor $2\pi R'$ in (5.1) comes from the trivial integration along the circle.

The interpretation of the twisted circle backgrounds (3.1) as freely acting orbifolds of $\mathbb{R}^9 \otimes S^1_{R'}$, divides the closed string spectrum (3.4) in twisted and untwisted sectors. The untwisted sector is composed of modes with $w = 0(\mathrm{mod} N)$. The $k^{th}$ twisted sector ($k = 1, \ldots, N-1$) contains modes with $w = k(\mathrm{mod} N)$. These modes will couple to linear combinations of field theory operators determined by $\gamma_t$, the matrix that represents the action of the orbifold on the Chan-Paton indices [26]. Since $\gamma_t$ for the twisted circle compactifications coincides with that of $\mathbb{C}^3/\mathbb{Z}_N$ orbifolds, we should consider the same linear combinations as in that case. This suggests that an scalar closed string mode with $w = k(\mathrm{mod} N)$ will couple to

$$W^{(k)} = \frac{1}{\sqrt{N}} \sum e^{2\pi i j \nu_k} \widetilde{W}^{(j)}(\nu_k) \, , \tag{5.2}$$

where $\widetilde{W}^{(j)}(\nu_k)$ is the operator (5.1) with the fields belonging to the $j^{th}$ gauge group factor and $s$ substituted by $(\nu_k + s) = wN^{-1}$.

In the noncommutative field theory limit [30], taken at fixed radius, the on-shell condition a closed string mode $\varphi$ with $w = k \pmod N$ is

$$\tilde{p}^2 + y^2 + (2\pi R')^2 (\nu_k + s)^2 + 8\pi^2\alpha' (N'_k + \bar{N}'_k) = 0 \, , \tag{5.3}$$

where we have used (3.4) and neglected terms suppressed by two powers of $\alpha'$. We have now all the ingredients to derive the contribution to the field theory effective action from a closed string exchange, following the analysis of section 2. Generalizing (2.7) and (2.11) to the present case, we obtain from the emission, propagation and posterior absorption of



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$\varphi$ by the brane

$$\Delta S = \frac{h_\varphi}{2\pi R'} \sum_{s=-\infty}^{\infty} \int \frac{d^3p}{(2\pi)^3} \frac{d^dv}{(2\pi)^d} \frac{W^{(k)}(P)\, W^{(N-k)}(-P)}{(\, v^2 + \tilde{p}^2 + (2\pi R')^2(\nu_k + s)^2\,)^{\frac{d}{2}+2}} e^{iyu}\,, \tag{5.4}$$

where $P \equiv (\tilde{p}, v, \nu_k + s)$, $d$ is the number of transversal directions where the closed string can propagate. $h_\varphi$ depends on the disk amplitude with one insertion of $\varphi$ and on the last term in (5.3).

All scalar closed string modes contribute to the field theory effective action as in (5.4), only differing in the value of $h_\varphi$. Since $W^{(k)}$ (5.3) carries no momentum along the circle, it can only couple to closed string modes with $p_y = 0$. In section 3 we saw that for these modes, the last term in (5.3) reproduces the spectrum in the $k^{th}$ twisted sector of the related $\mathbb{C}^3/\mathbb{Z}_N$ orbifold. We will assume that the disk amplitudes with one insertion of these modes and no open string insertions also coincide for orbifolds and twisted circles backgrounds. With this hypothesis, after taking into account the effect of the whole closed string tower, $h_\varphi$ should be promoted to the same quantities that govern the effective action of $D3$-branes in $\mathbb{C}^3/\mathbb{Z}_N$ orbifolds (2.11): $\epsilon_k/2\pi^2 c$, with $c$ given in (2.9) and (2.12).

We are ready to derive the contributions to the 2-point functions predicted by the closed exchange technique. We study first the non-planar contributions. They are obtained by expanding the Wilson line operators to first order in the fields. The linear combinations that diagonalize $\Pi_{ij}^{(M)}$ appear in a natural way: $B_M^{(k)} = \frac{1}{\sqrt{N}} \sum e^{2\pi i j \nu_k}\, \mathrm{Tr}\, Z_M^{(j)}$, with $Z_M^{(j)} = (A_\mu, A_y, \phi_a)^{jj}$. In order to compare with previous results, it is important to take into account that the fields in the exponent of (5.1) are $Z^{(j)}(x) = \frac{1}{2\pi R} Z_{n=0}^{(j)}(x)$, with $Z_{n=0}^{(j)}$ the zero-modes for which the 2-point functions were calculated in the past section. From (5.4) we obtain then an expression for $\Pi_k^{(M)}$ analogous to (4.9), with the functions $F_k^{(M)}$ replaced by

$$F_k^{(1)st} = 4R'^2 \sum_{s=-\infty}^{\infty} \frac{\tilde{p}^2}{(\,\tilde{p}^2 + (2\pi R')^2(\nu_k + s)^2\,)^2}\,, \tag{5.5}$$

$$F_k^{(2)st} = 4R'^2 \sum_{s=-\infty}^{\infty} \frac{(2\pi R')^2(\nu_k + s)^2}{(\,\tilde{p}^2 + (2\pi R')^2(\nu_k + s)^2\,)^2}\,, \tag{5.6}$$

$$F_k^{(3)st} = 2R'^2 \sum_{s=-\infty}^{\infty} \frac{1}{\tilde{p}^2 + (2\pi R')^2(\nu_k + s)^2}\,. \tag{5.7}$$

For the origin of the relative factor 1/2 in $F_k^{(3)st}$ see (2.12). Using (A.1) and (A.2), it is easy to show that $F_k^{(M)st} = F_k^{(M)}$. $\Pi_k^{a\mu} = \Pi_k^{ay} = 0$ because the associated integrand is an odd function of $y$. $\Pi_k^{y\mu} = 0$ is obtained by summing the contributions from the $k^{th}$ and $(N-k)^{th}$ sectors.

To derive the planar contribution to the 2-point functions from (5.4), we expand one Wilson line operator to second order in the fields while retaining from the second one only the leading term. Due to the structure of (5.4), we get only contributions diagonal in the group label indices, as was shown to be the case in the field theory (see figure 3). The leading term in the expansion of $W^{(k)}$ is proportional to $\frac{1}{\sqrt{N}} \sum e^{2\pi i j \nu_k} n_j \, \delta^{(3)}(p)$. Since the closed string propagator is finite as $\tilde{p} \to 0$ while the gauge field $A_\mu$ appears always multiplied by corresponding $\tilde{p}^\mu$ factors, the result for $\hat{\Pi}^{\mu\nu}_{ii}$ is zero. For the other components we get exactly (4.11).

The contribution to the 1-loop effective action from the vacuum diagrams, $\Gamma$, can be evaluated using that

$$\frac{\partial \Gamma}{\partial \beta} = -\frac{V_3}{2} \sum_\rho (-1)^{F_\rho} \int \frac{d^3 l}{(2\pi)^3} N_{\bar{\nu}_\rho} \, l \,, \tag{5.8}$$

where $\beta = 2\pi R$ and $V_3$ represents the volume of the three noncompact dimensions. The integration is over the euclidean 3-momentum and $\rho$ runs over all degrees of freedom of the theory. We obtain

$$\Gamma = \frac{V_4 \, T^4}{\pi^2} \sum_{i,j} n_i \, n_j \sum_k \epsilon_k \, \cos\left(2\pi(i-j)\nu_k\right) \, Z_4[\nu_k] \,, \tag{5.9}$$

with $V_4 = 2\pi R \, V_3$. Retaining only the first term in the expansion of $W^k$, one can check that (5.4) also reproduces $\Gamma$. It was argued in [1] that $\Gamma$ for the $\mathbb{C}^3/\mathbb{Z}_N$ quiver theories of section 2, could be recovered from (2.6) in an analogous way. However this implied identifying $\frac{1}{\tilde{p}}|_{\tilde{p}=0}$ with an UV cutoff $\Lambda$, and introduced certain ambiguity. This problem does not arise in the theories treated here, since they are UV finite.

Let us summarize our results. The closed string picture, remarkably, captures both the leading planar and non-planar terms which will give rise in the limit $T \to \infty$ to the UV and IR divergences linked to the absence of supersymmetry. Moreover, it offers an explanation for why the same coefficients $\epsilon_k$ appear in $\mathbb{C}^3/\mathbb{Z}_N$ orbifolds and twisted circle backgrounds. Twisted circle backgrounds, for sufficiently large radius, cure the problem of closed string tachyons by shifting the spectrum with a positive winding energy. However this affects equally NSNS and RR sectors. Since NSNS and RR sectors contribute with opposite sign to (5.4), this shift is not perceived by the field theory. The sign of the leading UV/IR mixing effects is still given by that of the gap between the lowest NSNS and RR modes.

The nonplanar functions that we calculated in this and the past section are regular in $\tilde{p}$. From the string point of view, this crucially depends on being in a regime where winding energy dominates the closed string spectrum. Instabilities arising from finite UV/IR effects can be expected to have a qualitatively different nature from those associated to IR divergences. It is then an open question whether the one to one correspondence between tachyons and noncommutative instabilities found for $\mathbb{C}^3/\mathbb{Z}_N$ orbifolds, might hold in more general IR divergent cases. The fact that the nonplanar polarization tensor (4.9) develops a pole-like behavior at $R = 0$, value which maps to the string regime with tachyons, supports this possibility. The twisted circle backgrounds provide us with another field theory related to an string regime with tachyons, that on the world-volume of a $D2$-brane placed transverse to the circle. In the next section we will analyze the leading UV/IR mixing effects in this theory.

## 6. Transversal $D2$-branes

We consider $n$ Type IIA $D2$-branes extending in the directions transverse to (3.1) and localized at $z^l = y = 0$. It is convenient to picture the brane configuration on the $\mathbb{R}^{10}$ covering space. It corresponds to an infinite array of $D2$-brane sets placed at intervals $2\pi R$ on the $y$ direction. Let us denote open strings stretching between the $i^{th}$ and $j^{th}$ sets by $(i,j)$. The action (3.1) relates $(i,j)$ strings with $(i+1,j+1)$, but does not impose any projection on each of them separately. Thus the spectrum on the branes will be maximally supersymmetric [33]. We are interested in taking the field theory limit such that the masses of the winding strings are keep finite, i.e. $\alpha' \to 0$ with $R/\alpha' = r$ fixed. The field content on the branes will consist of an infinite set of multiplets $\Psi_w$ with masses $m_w = wr$. Each multiplet $\Psi_w$ transforms in the adjoint of $\mathrm{U}(n)$ and contains 3 complex scalars $\phi_{l,w}$, 4 Weyl fermions $\psi_{\alpha,w}$ and, for $w \neq 0$, a massive vector $A_{\mu,w}$, while for $w = 0$ a massless vector plus a real scalar $A_{\mu,0}$, $\varphi_0$. The scalars $\phi_{l,0}$, $\varphi_0$ are associated to fluctuations of the branes in the $z^l$ and $y$ directions. The previous field theory limit corresponds to the string regime with tachyonic modes, as can be seen from (3.6).

In this section we will focus on the evaluation of the polarization tensor. We turn on a B-field on the two spatial directions of the $D2$-branes. We will analyze first what the closed string picture predicts for the leading UV/IR mixing effects. As before, closed strings will couple to open Wilson line operators. The explicit construction of these operators is again involved. For our purposes, it will be enough to know few terms in their expansion

$$W^{(w)}(p) = \delta_{w,0}\,(2\pi)^3\delta^{(3)}(p)\,\mathrm{Tr}\,\mathbb{1} + ig\tilde{p}^{\mu}\,\mathrm{Tr}\,A_{\mu,w}(p) + \cdots\,. \tag{6.1}$$

Given that the identity operator carries no winding number, it can only appear in the expansion of $W^{(0)}$.

Closed string modes that couple to (6.1) have winding number $w$. Their momentum $p_y$ is a free parameter, since the $D2$-branes break translational invariance along the $y$ direction. There is however a restriction. $D2$-branes located at $z^l = 0$ do not break rotational invariance on the $C^3$, and thus their vacuum state carries no angular momentum. Neither do the fields $A_{\mu,w}$. Therefore (3.2) implies $p_y = m/R$ with $m \in \mathbb{Z}$.

In the noncommutative field theory limit that we are considering in this section, the on-shell condition for a closed string mode $\varphi$ with $w = k \pmod N$ and $p_y = m/R$ reads

$$\tilde{p}^2 + y^2 + \frac{4\pi^2 m^2}{r^2} + 8\pi^2\alpha'(N_k' + \bar{N}_k') = 0\,, \tag{6.2}$$

where again we have neglected terms suppressed by two $\alpha'$ powers. Using the same reasoning as in the past sections, we obtain the following contribution to the transversal part of the polarization tensor from a closed string exchange

$$\Pi_w^{\mu\nu} \sim h_\varphi\, r^{-1} g^2 \frac{\tilde{p}^\mu\tilde{p}^\nu}{\left(\tilde{p}^2 + \frac{4\pi^2m^2}{r^2}\right)^2}\,. \tag{6.3}$$

Notice that the dependence on $\tilde{p}$ is that proper of a 4d theory. This arises as follows. The definition of the closed string propagator includes now a factor $1/2\pi R$, because the brane is transversal to the circle. In the $\alpha' \to 0$ limit that keeps finite the mass of winding strings, $R$ absorbs one $\alpha'$ factor and gives raise to the $r^{-1}$ factor in (6.3). As a result, the counting of $\alpha'$ powers is as for the $D3$-brane case.

The combination $2\pi m/r$ in (6.3) plays an analogous role to the winding energy in (5.4). There is however a crucial difference; in the latter case, the winding energy could not be zero if $w \neq 0(\text{mod}N)$. Contrary, $m = 0$ is an allowed value for any $w$. It is clear that $m = 0$ will dominate the small $\tilde{p}$ limit of the sum in (6.3). Let us focus then on this value.

The quantity $h_\varphi$ depends on the last term in (6.2). When $m = 0$, the associated closed string mode has $p_y = 0$. As we have already explained, the last term in (6.2) reproduces then the spectrum of the $k = w(\text{mod}N)$ twisted sector of the related $\mathbb{C}^3/\mathbb{Z}_N$ orbifold. $h_\varphi$ also depends on a numerical factor, $D^{tc}_\varphi$, which can be extracted from the disk amplitudes of $\varphi$ with boundary conditions on the $D2$-brane. However we can not take now the brane in its vacuum state, since this restricts us to $w = 0$. Let us consider the disk amplitude with one insertion of $A_{\mu,w}$ on its boundary. This should reproduce $D^{tc}_\varphi$ times the second term in (6.1). As in the past section, let us assume that $D^{tc}_\varphi$ coincides with the analogous factors that appeared in the orbifold case of section 2. In that case, after taking into account the contribution from all such modes to (6.3), $h_\varphi$ should be promoted to the same quantities $\epsilon_k$ that appeared in the previous sections. Hence, at small $\tilde{p}$, the closed string picture suggests the following result for the polarization tensor

$$\Pi^{\mu\nu}_w \sim \epsilon_k \, r^{-1} g^2 \frac{\tilde{p}^\mu \tilde{p}^\nu}{\tilde{p}^4} \, . \tag{6.4}$$

with $k = w \pmod N$.

We will like to add a comment. Comparison with (2.10), for $w = 0$, implies that the expansion of $W^{(0)}$ at leading order in the fields should include a term $\sim \alpha' p_y \operatorname{Tr} \varphi_0 = \frac{m}{r} \operatorname{Tr} \varphi_0$, where $\varphi_0$ is the massless scalar of the $\Psi_0$ multiplet. For $W^{(w)}$, with $w \neq 0$, we expect that this is promoted to a term $\sim \frac{m}{r} \operatorname{Tr} A_{L,w}$, where $A_{L,w}$ denotes the longitudinal component of the massive gauge field $A_{\mu,w}$. Using this and the closed string exchange technique, we could try to derive a contribution to the longitudinal piece of the polarization tensor. However the string derivation predicts no contribution from $m = 0$, and thus we have ignored it. This is consistent with the field theory analysis below.

We will confirm now the proposal (6.4) from a direct field theory calculation. The appearance of IR divergences could have surprised us, since the field content of the $D2$-brane theory is supersymmetric. Notice that the spectrum on $D3$-branes, both at orbifold singularities and twisted circles, was not supersymmetric, and the coefficients $\epsilon_k$ were essentially counting number of bosonic minus fermionic degrees of freedom from the point of view of the field theory. Contrary to these cases, supersymmetry is broken by interactions on the $D2$-brane. Thus we need to know how the Feynman rules get modified.

Let us come back to image of the $D2$-brane configuration on the covering space. We will denote as $Z_{(i,j)}$ a field arising from an string stretching between the $i^{th}$ and $j^{th}$ $D2$-brane sets. Before imposing (3.1), our gauge theory is $\mathrm{U}(\infty)$ broken to an infinite product of

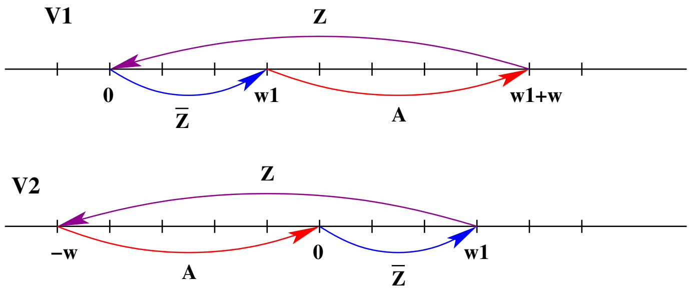


**Figure 4:** Interaction vertices $V_1$ and $V_2$ in the covering space.

$\mathrm{U}(n)$ factors by the separation of the branes. A field $Z_{(i,j)}$ transforms in the fundamental representation of the gauge group factor $i^{th}$ and antifundamental of the $j^{th}$. Therefore the only terms that can appear in the lagrangian of the theory form closed paths in the direction $y$; for example[5]

$$\mathrm{Tr}\, \bar{Z}_{(i,j)} Z_{(j,i)}\,, \qquad \mathrm{Tr}\, Z^1_{(i,j)} Z^2_{(j,k)} Z^3_{(k,i)}\,. \tag{6.5}$$

All the kinetic terms are canonically normalized and the three- and four-point vertices depend on a single coupling constant $g$.

Invariance under (3.1) implies

$$Z_{(i,j)} = \omega_Z^i \; Z_{(0,j-i)}\,, \tag{6.6}$$

with $\omega_{\phi_l} = e^{2\pi i \frac{b_l}{N}}$, $\omega_{\psi_\alpha} = e^{2\pi i \frac{a_\alpha}{N}}$ and $\omega_{A_\mu} = 1$. From now on we will take the fields $Z_{(0,w)}$ as reference and we will label them simply as $Z_w$. They form the multiplets that we defined in the beginning of the section as $\Psi_w$.

In order to obtain the polarization tensor for the vector fields $A_{\mu,w}$, we have to evaluate the graphs in figure 2 and 3. We need to know the propagators and vertices of the theory in terms of the fields $Z_w$. Since $\bar{Z}_{(i,j)} Z_{(j,i)} = \omega_Z^{-w} \bar{Z}_{-w} Z_w$, with $w = i - j$, and the field $Z_{(i,j)}$ is canonically normalized, the propagator for the field $Z_w$ will carry an additional factor $\omega_Z^w$. For constructing the graphs in figure 2 and 3, two vertices are relevant

$$V_1 = g_1 \;\, \mathrm{Tr}\, \bar{Z}_{w_1} A_{\mu,w} Z_{w_2}\,, \qquad V_2 = -g_2 \;\, \mathrm{Tr}\, \bar{Z}_{w_1} Z_{w_2} A_{\mu,w}\,, \tag{6.7}$$

with $w_1 + w_2 + w = 0$ and $g_1$, $g_2$ the coupling constant associated to each vertex. Using (6.6), it is easy to see that $g_1 = g\, \omega_Z^{-w_2}$ and $g_2 = g\, \omega_Z^{w_1}$. The different powers of $\omega_Z$ associated to the two vertices may result surprising. However we should notice that the two vertices come from different configurations in the covering space, see figure 4.

The non-planar graph of figure 2 arises when the two vertices are $V_i$, $V_j$ with $i \neq j$ $(i, j = 1, 2)$. Using the rules explained above, we obtain the following contribution to the non-planar polarization tensor from a field $Z$ running in the loop

$$\frac{1}{2} \left( \omega_Z^w + \omega_Z^{-w} \right) \Pi_Z^{\mu\nu}\,, \tag{6.8}$$

[5]These terms contain also derivatives. We are being schematic for simplicity.

where $\Pi_Z^{\mu\nu}$ would be the analogous result when $\omega_Z = 1$. This simpler situation corresponds to $D2$-branes placed transversally to an ordinary circle, and is T-dual to $D3$-branes wrapping an ordinary circle of radius $r^{-1}$. The evaluation of 1-loop integrals is most convenient in the T-dual picture. It results in a piece which is $r$-independent and reproduces the uncompactified $r^{-1} \to \infty$ result, and a part that is UV finite and $r$ dependent. The former one gives rise to an IR divergence and dominates in the limit of small $\tilde{p}$. Since this is the IR divergence of a 4d theory, we have[6] $\Pi_Z^{\mu\nu} \sim r^{-1} g^2 \frac{\tilde{p}^\mu \tilde{p}^\nu}{\tilde{p}^4}$, with $r^{-1}g^2$ the $4d$ coupling constant. Summing now to $Z = (\phi_l, \psi_\alpha, A_\mu)$ and having into account the phases in (6.8), the parameters $\epsilon_k$ reappear and we recover (6.4).

It is also interesting to evaluate the planar piece of the polarization tensor, figure 3. It arises when both vertices are $V_1$ or $V_2$. In these cases, the dependence on the phases $\omega_Z$ cancels. Since the spectrum of the theory is supersymmetric, adding the contributions from all fields will give a vanishing result. For the same reason the 1-loop vacuum energy of the theory is zero. Both results are predicted by the closed string picture. Notice that the vacuum diagrams can only come from the leading term in the expansion of $W^{(0)}$, and the planar part of the polarization tensor from higher terms in the expansion of $W^{(0)}$, which we have not written explicitly in (6.1). Since $\epsilon_0 = 0$, we obtain no contribution.

## 7. Conclusions

The main motivation of this paper was to answer some of the open questions left in [1], where a direct relation between noncommutative instabilities and closed string tachyons was found for a family of gauge theories on $D3$-branes at $\mathbb{C}^3/\mathbb{Z}_N$ orbifolds. In this section we summarize our results.

The first question was whether the relation between noncommutative instabilities and closed string tachyons holds in other examples. A useful tool to study this problem was a proposal to derive UV/IR mixing effects purely in the language of closed strings. This point of view supported the distinction of two separate pieces in the leading UV/IR mixing effects: their functional dependence on $\tilde{p}$ and their sign. The sign determines if the leading UV/IR mixing effects will tend to destabilize or not the theory. The functional dependence on $\tilde{p}$ can be finite or divergent in the IR depending if supersymmetry is or not restored at some high scale.

The closed string derivation suggested that the sign of the leading UV/IR mixing effects is linked to the misalignment between the NSNS and RR towers of the parent string theory. In order to check this proposal, we have analyzed in this paper twisted circle compactifications of Type II string theory. The NSNS and RR towers are shifted in the same amount by a positive winding energy. Tachyonic modes appear only for twisted circles of sufficiently small radius. We have studied Type IIB $D3$-branes wrapped on the twisted

[6] In addition to the graphs in figure 2, there are 1-loop diagrams with the insertion of a four-point vertex contributing to the nonplanar polarization tensor. The same considerations above can be applied to them with similar results.



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circle and Type IIA $D2$-branes transversal to it. We showed that in both cases the UV/IR mixing signs are governed by the (mass)$^2$ difference between the lowest NSNS and RR modes, and thus does not perceive the additional winding energy.

The appropriate field theory limits to $D2$- and $D3$-branes correspond to string regimes with and without tachyons respectively. Although the signs of the leading UV/IR mixing effects were not able to distinguish both regimes, their functional dependence on $\tilde{p}$, which we could related to a closed string propagator, was. We found IR finite corrections for $D3$-branes and IR divergent ones for $D2$-branes. Therefore the example of the $D2$-branes provides further support for a correspondence between closed string tachyons and non-commutative instabilities associated to IR divergences. For $D3$-branes a weaker but still interesting result holds. UV/IR mixing effects are still governed by the lowest modes in the associated closed string sector. Moreover destabilizing UV/IR mixing effects are absent for those sectors related to closed strings which are stable for any radius of the twisted circle.

For the noncommutative gauge theory on $D3$-branes both at orbifolds and twisted circle backgrounds, the coefficients that governed the leading UV/IR effects could be directly derived from the matter content of the theory. It is worth stressing that this is not the case for the theory on the transversal $D2$-branes, in which the spectrum is supersymmetric while supersymmetry is broken by interactions. Yet the leading UV and IR behavior of the field theory can be understood in terms of closed strings.

Since the severe consequences of UV/IR mixing were first recognized in [5], the different behavior of the planar and nonplanar sectors of the theory has been a puzzling fact. The planar sector can have UV divergences, while the nonplanar translate UV into IR behavior. Cancellation of the leading UV divergences for supersymmetric theories implies the absence of the leading IR divergences. This suggests that planar and non-planar terms are not disconnected but they are part of a unified structure. In this line, we have shown that the gauge invariant effective action containing the leading UV/IR mixing effects, reproduces also the leading contribution to the planar graphs. The 1-loop vacuum graphs and planar 2-point functions can be obtained from the actions (5.4) and (6.3) for the $D3$- and $D2$-brane theories respectively, when at least one of the open Wilson line operators is substituted by its component in the identity.

For the gauge theory on the $D3$-branes, both planar and nonplanar pieces are finite. We can then smoothly send the noncommutativity parameter $\theta$ to zero. The action (5.4) does not trivialize in this limit. In the absence of noncommutativity it might look surprising that the closed exchange technique is able to reproduce some pieces of the effective action. This however points towards the fact that the field theory limit retains relevant information about closed strings. We regard this as a nontrivial implication of our results.

In [12] it was shown that the closed exchange technique could also reproduce the logarithmic IR divergences of 4d noncommutative gauge theories. However no string interpretation was given of the coefficients governing these divergences. It will be very interesting to determine whether a relation with closed string properties also holds for subleading UV/IR mixing effects.



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## A. Some field theory results

Let us consider a field with momentum $p_y = (m - \bar{\nu})/R$ ($m \in \mathbb{Z}$) along a circle of radius $R$, with $\bar{\nu}$ arbitrary. We define $N_{\bar{\nu}}$ as

$$\sum_{m=-\infty}^{\infty} \frac{1}{(m-\bar{\nu})^2 + R^2 p^2} = \frac{\pi}{Rp} N_{\bar{\nu}}(Rp) \,. \tag{A.1}$$

For bosons with integer moding along the circle ($\bar{\nu} = 0$), $N_0 = 1 + 2n_B$, with $n_B$ the Bose-Einstein distribution; for fermions with half-integer moding ($\bar{\nu} = 1/2$), $N_{\frac{1}{2}} = 1 - 2n_F$, with $n_F$ the Fermi-Dirac distribution. For arbitrary $\bar{\nu}$ we can consider $\frac{1}{2}(-1)^F(N_{\bar{\nu}} - 1)$, with $F$ the fermion number, the generalization of $n_B$ and $n_F$. The sum (A.1) can be evaluated in a closed form, with the result

$$N_{\bar{\nu}}(x) = \frac{\sinh 2\pi x}{\cosh 2\pi x - \cos 2\pi\bar{\nu}} \,. \tag{A.2}$$

In general $\frac{1}{2}(-1)^F(N_{\bar{\nu}} - 1)$, contrary to $n_{B,F}$, is not always positive and thus it can not have the interpretation of occupation numbers. This is not a problem, since $y$ will be always an spatial direction for us.

We focus next in a $4d$ euclidean noncommutative U(1) theory with one direction compactified on a circle. The matter content will consist of two real scalars in the adjoint representation: $\varphi$, with arbitrary $\bar{\nu}$, and $\phi$, with $\bar{\nu} = 0$ and couplings as if obtained from the dimensional reduction of the gauge field from 5 to 4 dimensions. We want to derive the contribution to the polarization tensor of the gauge field and self-energy of $\phi$, from the 1-loop exchange of $\varphi$. We are only interested in the contributions that arise from high loop momenta. These are given by

$$\begin{aligned} \bar{\Pi}^{AB} &= 4g^2 \frac{1}{2\pi R} \sum_m \int \frac{d^3 l}{(2\pi)^3} \frac{2L^A L^B - \delta^{AB} L^2}{L^4} \sin^2 \frac{\tilde{p}.l}{2} \,, \\ \bar{\Sigma} &= -4g^2 \frac{1}{2\pi R} \sum_m \int \frac{d^3 l}{(2\pi)^3} \frac{1}{L^2} \sin^2 \frac{\tilde{p}.l}{2} \,, \end{aligned} \tag{A.3}$$

where the indices $A, B$ run over the non-compact directions $\mu = 1, 2, 3$ and the circle; $L = (l_\mu, l_y)$, with $l_y = (m - \bar{\nu})/R$, and the external momentum is $P = (p_\mu, p_y)$. For $p_y \neq 0$, (A.3) does not fulfills the Ward identities and subleading contributions must be taken into account. In the following we limit ourselves to $p_y = 0$.

Using (A.1), we obtain

$$\bar{\Pi}^{\mu\nu} = 2g^2 \int \frac{d^3 l}{(2\pi)^3} \left[ \left( \frac{N_{\bar{\nu}}}{l} - \frac{dN_{\bar{\nu}}}{dl} \right) \hat{l}^\mu \hat{l}^\nu - \frac{N_{\bar{\nu}}}{l} \delta^{\mu\nu} \right] \sin^2 \frac{\tilde{p}.l}{2} \,,$$

$$\bar{\Pi}^{yy} = 2g^2 \int \frac{d^3 l}{(2\pi)^3} \frac{dN_{\bar{\nu}}}{dl} \sin^2 \frac{\tilde{p}.l}{2} \,,$$
$$\bar{\Sigma} = -2g^2 \int \frac{d^3 l}{(2\pi)^3} \frac{N_{\bar{\nu}}}{l} \sin^2 \frac{\tilde{p}.l}{2} \,, \tag{A.4}$$

where $\hat{l}^\mu = l^\mu / l$. Using $\sin^2(x) = \frac{1}{2}(1 - \cos 2x)$, we can separate the planar and nonplanar pieces in the previous expressions. The integration over the angular variables can be performed in an straightforward way, with the following results for the non-planar part

$$\Pi^{\mu\nu} = \frac{\tilde{p}^\mu \tilde{p}^\nu}{\tilde{p}^2} \frac{g^2}{2\pi^2 \tilde{p}} \int_0^\infty dl N_{\bar{\nu}} \left( \sin \tilde{p}l - \tilde{p}l \cos \tilde{p}l \right) , \tag{A.5}$$
$$\Pi^{yy} = \frac{g^2}{2\pi^2 \tilde{p}} \int_0^\infty dl N_{\bar{\nu}} \left( \sin \tilde{p}l + \tilde{p}l \cos \tilde{p}l \right) , \tag{A.6}$$
$$\Sigma = \frac{g^2}{2\pi^2 \tilde{p}} \int_0^\infty dl N_{\bar{\nu}} \, \sin \tilde{p}l \,. \tag{A.7}$$

The planar part is

$$\hat{\Pi}^{\mu\nu} = 0 \,, \qquad \hat{\Pi}^{yy} = 2\hat{\Sigma} = -\frac{g^2}{\pi^2} \int_0^\infty dl N_{\bar{\nu}} \, l \,. \tag{A.8}$$

The expressions above apply when the field circulating in the loop is a real scalar. For a general field with fractional momentum $\bar{\nu}$, we should multiply (A.5)–(A.8) by the number of degrees of freedom of the field times $(-1)^F$, with $F$ its fermion number. When there are several adjoint scalars, the self-energies are $\Sigma^{ab} = \delta^{ab} \Sigma$ with $\Sigma$ as in (A.5)–(A.8). When there are several group factors, the non-planar 2-point functions will mix different group labels if there are bifundamental fields running in the loop. Each bifundamental degree of freedom gives half the contribution in (A.5)–(A.7). Since (A.5)–(A.8) only depend on the components of $p_\mu$ along the noncommutative plane, the Wick rotation along the additional noncompact direction to Minkowski signature is trivial.

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